Central Bank Digital Currency intermediation*

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This paper investigates the effects of Central Bank Digital Currencies (CBDC), focusing particularly on the intermediary role of commercial partners. Existing literature on CBDC has primarily analysed the impact of digital currencies as a substitute for bank deposits and physical cash. This research, on the other hand, examines the implications of assigning to commercial partners the responsibilities of customer screening and of maintaining CBDC wallets under the constraint that CBDC holdings are non-interest-bearing. This no-interest feature, highlighted in recent central bank projects, restricts commercial partners' revenue opportunities from interest rate spreads and can impact economic incentives to facilitate CBDC transactions. The findings of this paper show that under a no-interest constraint, CBDC deposits become less appealing to households, amplifying income volatility and diminishing the resources available to commercial partners' activities.

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1 Introduction

According to some CBDC projects, commercial partners would be responsible for managing clients' CBDC wallets and monitoring policies on behalf of central banks. In addition, the more advanced CBDC projects consider that the digital currency would not bear any interest, therefore not allowing commercial partners to benefit from any spread between assets and liabilities held in CBDC (Bank of England, 2020; ECB, 2023). These Central banks' efforts to issue Central Bank Digital Currencies (CBDC) have also sparked the attention of recent literature aiming to understand the impact of CBDC on the economy. The main focus is to try to explain the potential impact of CBDC on bank intermediation and stability by analysing their substitutability with respect to bank deposits or physical cash.

This paper uses the model of Cúrdia and Woodford (2016) to study how the proposed designs for CBDC affect the provision of digital currency considering the responsibilities attributed to the commercial partners. One can consider the commercial partner as a wallet provider responsible for screening potential clients for additional services authorized by the central bank and running standard "know-your-customer" policies in line with anti-money-laundry practices.[1] These responsibilities incur a cost for the provider and, for the arrangement to be economically interesting, they should be remunerated in some way. To the best of my knowledge, this is the first paper that has given specific attention to the intermediary problem. It is also the first paper to consider one of the main current policy constraints in CBDC, namely the absence of interest bearing on deposits held in the form of CBDC.

To do so, I adapt the problem of the financial intermediary in Cúrdia and Woodford (2016) to consider that the effort exerted by the partner (henceforth called wallet provider) affects the probability of accepting a bad client. With a certain probability,

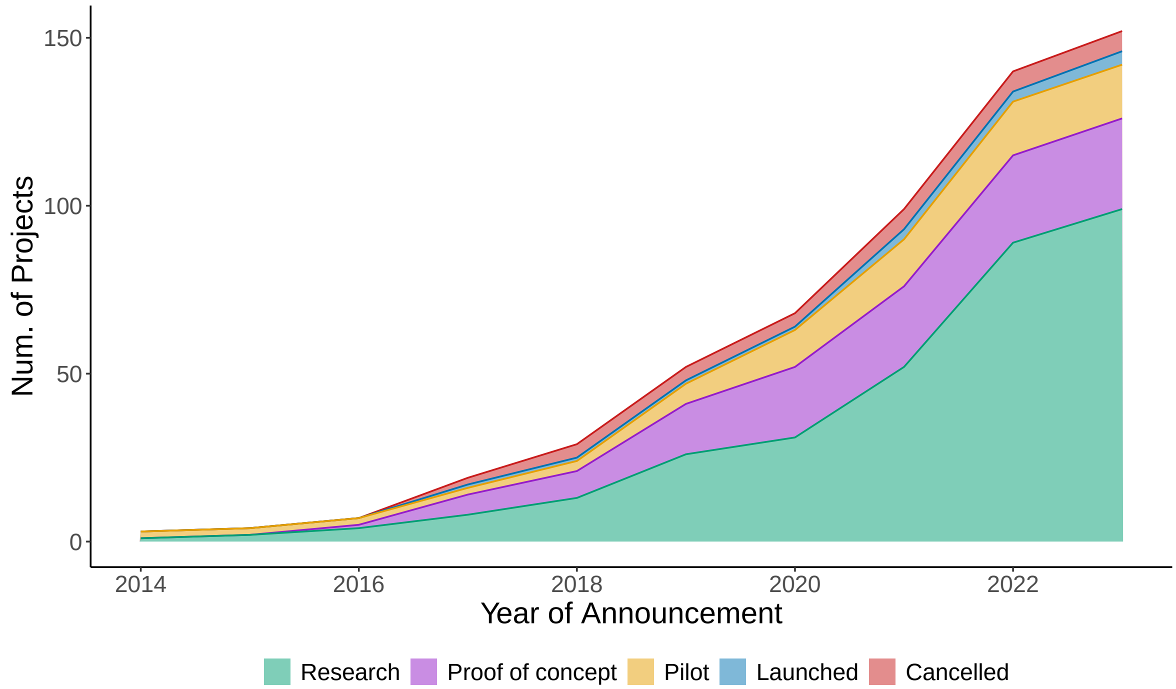
[1] These services are in addition to the deposit provision and one can consider, for instance, schemes that allow households to pay for consumption in instalments.

this client can misuse its CBDC funds and generate a cost to the wallet provider. This cost, which is proportional to the clients' funds, can be considered to be an actual fraud cost or a reputational cost demanding a reprimand to the provider. The main distinction with respect to Cúrdia and Woodford (2016) is that effort now has a secondary benefit, which is the reduction of potential bad clients. I then analyse the dynamics of different shocks under this setup, comparing the model specification with and without the interest rate constraint in the CBDC deposits.

By restricting deposits to bear no interest rate, the central bank induces more volatility in households' income after a shock. This is caused by the fact that deposits in CBDC become less attractive to patient households, also reducing available resources to commercial partners to generate financing services. Shocks to preferences that increase current consumption also cause more prominent movements in income, while the service fees are less reactive when compared to a scenario of interest-bearing CBDC. In the absence of financial frictions and no interest rate in CBDC deposits, the services provided by the wallets tend to deviate for a longer period from their steady state, given that some adjustment mechanisms are not present.

This paper relates to previous work on CBDC by analysing the implementation of digital currency and how it shapes cycle dynamics. It is closely associated with the works of Bacchetta and Perazzi (2022), Ferrari et al. (2022),Fernández-Villaverde et al. (2021) and others, but also contributes by shifting the attention to the wallet. The aforementioned works also allow for CBDC to behave as interest rate-bearing deposits, which I do not.

The paper is organised according to the following structure. I present a literature review regarding CBDC in section 3. The model used for this analysis, as well as the main distinction from Cúrdia and Woodford (2016), are presented in section 4. Finally, I present the simulation results in section 5.



Source: CBDC tracker

Figure 1: Accumulated evolution of the number of projects in CBDC initiated by central banks.

2 A background on CBDC

CBDC started to be idealised in their current format around 2013-14, with the Central Bank of Uruguay and the People's Bank of China being the first monetary authorities to disclose research on this type of digital currency.[2] The primary idea of these projects and the ones that followed is to create a digital form of the countries' fiat currency, which could be used in online and offline transactions. Figure 1 shows the evolution of CBDC projects run by the central banks around the world. It is possible to notice that there was a pickup in research pace regarding CBDC from 2020 to 2022.

In this sense, CBDC would have features of both deposits with commercial banks

[2]The Bank of Finland operated a pre-paid smart card during the 1990s which could be considered as a primitive form of CBDC (Grym et al., 2020). The system was then shut down after debit and credit cards became less expensive.

– but wouldn't be susceptible to potential destruction by bank runs – and cash, but potentially bearing interest. Furthermore, in the process of conceiving the CBDC the monetary authorities face some technical decisions regarding its design, as explored next.

Token vs. Account-based

One of the main features is how to verify the validity of the transaction. This process can be either at an account-based level, where the transaction is validated by checking the individual's account balance, or at a token-based level, where validation is made by confirming the authenticity of the (digital) token. In both scenarios, central banks would hold a ledger that would allow them to (at least partially) observe the transactions using CBDC (Bank for International Settlements, 2021).

Wholesale vs. Retail CBDC

Another design decision that central banks face regarding CBDC is who would have access to the currency. If CBDC were to be used only by financial intermediaries, which are agents that already hold reserve accounts with the central bank, this digital currency would be denominated "wholesale CBDC". The main purpose of this currency would be inter-institution settlements and payments. On the other hand, if CBDC are used by the general public for day-to-day transactions, this would be a "retail CBDC".

The implementation of a wholesale CBDC most likely binds its design to an account-based option and might be considered a smaller paradigmatic change. Bank reserves with the central banks are already digital forms of payments that are backed by central banks' assets, hence the implementation of a wholesale CBDC would benefit with a potential improvement in the efficiency of already existing functionalities. Retail CBDC, however, is a more interesting change to the status quo. Retail CBDC is more flexible concerning the verification of a transaction (which could be either token or account-based) and creates a digital asset for households that is safer than traditional deposits – being backed by central banks' assets(Schumacher, 2024).

Other design-dependent features

Other features that are equally relevant to the design of CBDC are related to its cross-border usage and level of anonymity. Both are subject to the choice of central banks and can affect the desirability of the digital currency.

Concerning cross-border usage, CBDC can be a cheaper way to make international remittances or be used outside their issuing country, depending on the interoperability of the central banks' systems. This can bring efficiency gains, especially for international trade (Schumacher, 2024).

Regarding anonymity, this is a sensible topic that affects exclusively retail CBDC users. A token-based CBDC can provide high anonymity, without requiring identification for access. An account-based CBDC, however, will require some sort of identification to access and has lower to no anonymity, with privacy and confidentiality being protected by design and general data protection acts. Another thing to take into consideration is that, even under full anonymity, central banks must comply with anti-money laundry and know-your-customer policies.

3 Literature review

The body of literature focusing on Central Bank Digital Currencies (CBDC) has seen expressive growth in the past years, mainly fueled by the desire of many central banks to issue a digital version of their fiat currency (Boar and Wehrli, 2021). For instance, the Bank of England (BoE) and the European Central Bank (ECB) are two examples of central banks actively working towards designing and implementing a CBDC (HM Treasury, 2023; ECB, 2023). Nevertheless, it is not a common agreement that CBDC will generate more benefits than costs. In the USA, a relevant example is the recent objection by the House of Representatives with the "CBDC Anti-Surveillance State Act". The bill which was already passed in the House of Representatives and still needs to be voted by the Senate but, if approved, it amends the Federal Reserve Act to ban the Fed System from maintaining individuals' deposit accounts and the Federal Open Market Committee (FOMC) from using CBDC as a monetary policy tool.[3]

The work that closest tackles the CBDC's intermediation dynamics is Fernández-Villaverde et al. (2021), which uses the model by Diamond and Dybvig (1983) to explore the role of maturity transformation of CBDC. In their paper, the authors argue that while central banks can provide individuals' CBDC deposits, they do not have the technology or skill to invest these resources in long-term projects. For that, they rely on investment banks to it on their behalf. The authors also argue that central banks have two elements in their favour: debt seniority, generating higher returns, and the fact that they can not face forced liquidation, which allows them to limit withdraws.

The main findings under this setup are that central banks become deposit providers with more stability than commercial banks. Households internalise this fact *ex-ante*, driving out the latter and making the former a sole monopolist in the deposit market. However, under this context, the socially optimum amount of maturity transformation is still obtainable, as long as the central bank is allowed to compete against commercial banks for deposits.

CBDC implementation could also be analysed by following the New Monetarist Literature (NML) (Lagos and Wright, 2005). In their model, the digital currency is used to facilitate anonymous and centralised trade between agents in different markets. The digital currency could be used in one or more of these markets.

Davoodalhosseini (2021) uses this framework to analyse the impact of monetary policy under different CBDC contexts. According to the author, CBDC entails a certain

[3]The text of the bill can be accessed via https://shorturl.at/p0gRF

cost to final users.[4] However, in terms of monetary policy, central banks can better implement the policy, if they can observe the distribution of digital currency among users, by creating transfers that are contingent on the balances of agents and can improve welfare. However, they face a trade-off of imposing higher costs of handling CBDC. For cash users, the policy is implemented by evenly distributing newly created money. Similarly, Keister and Sanches (2023) considers credit constrained banking sector to analyse how the equilibrium allocations change when CBDC competes with bank deposits as a means of exchange. The authors argue that a CBDC can cause bank disintermediation, especially if considering a digital currency that bears interest, caused by the decrease in the liquidity premium.[5] However, the authors show that overall welfare is improved by increasing efficiency in the production of goods that can be bought using digital currency.

Another alternative to analyze the implementation of CBDC is to use models from the Industrial Organization literature, as explored by Agur et al. (2022),Whited et al. (2022) and Hemingway (2024).

Agur et al. (2022) suggest that households can be modelled as facing a Hotelling linear city, where they aim to minimize the distance between forms of money and their preferences for anonymity and payment security. Cash represents the anonymity extreme, while deposits represent the payment security extreme and when designing the digital currency the monetary authority can choose to position CBDC at any point between these extremes. The authors' results are analogous to previous work, with the trade-off between bank disintermediation and efficiency improvement being the main point discussed.

[4]These are costs associated with using the technology for the digital coin, e.g. digital wallets and accounts (Davoodalhosseini, 2021).

[5]It is important to notice that even though some of the works published so far consider an interesting bearing CBDC, most of the central banks or CBDC already implemented have opted to not allow their digital currency to do so. These are the cases of the e-Krona in Nigeria, the Sand Dollar in the Bahamas, the digital Pound in the UK and the digital Euro.

Whited et al. (2022), on the other hand, look at how the benefits of each asset affect the decision of the household to allocate wealth. In a model where CBDC competes with cash, bonds and bank deposits, each individual will allocate their endowed wealth to the asset with the higher (subjective) yield. The authors show that CBDC has the potential to crowd out bank deposits, but has a limited impact on lending when banks can switch to external funding. The decision to switch the funding depends on the frictions the banks face.

Hemingway (2024) models the implementation of the digital currency using a Salop circle problem. Banks compete in prices and are equally scattered around a circle with a continuum of depositors. When implemented, the CBDC would compete with the banks for deposits. The author argues that the introduction of CBDC, in the short term, causes a reduction of bank intermediation, with banks leaving the market. In the long run, this causes a higher concentration and market power on the remaining banks, which can push deposit rates down. Therefore, in the long run, the banks that stay in the market face lower cost pressure from competition with the CBDC.

Finally, a common approach that the CBDC literature uses to evaluate the impact of implementing a digital currency is by using New Keynesian models with assumptions about its substitutability vis-à-vis other assets. Bank competition setup is also a relevant assumption that generates significantly different results across published works.

Ikeda et al. (2020), for instance, analyse the impact of a digital currency considering its role as a unit of account. The authors find that in a two-country, open economy cashless model if a dominant currency issues a digital currency (a digital dollar in their example), monetary policy has a weak capacity to affect the economy.[6] This effect is more expressive in smaller economies with a higher share of imported goods consumption. Analogously, Ferrari et al. (2022) show that the implementation of a CBDC

[6] Following Goldberg and Tille (2016), a dominant currency is the one predominantly used for pricing of imports and exports invoices.

creates a non-arbitrage condition that generates a stronger "interest rate parity" in an open-economy context. The issuing country would "export" their monetary policy, making the non-issuing country more exposed to external shocks. Under a somewhat opposing argument, Bacchetta and Perazzi (2022) show that CBDC have the potential to generate more benefits than costs. The authors use an open-economy model with imperfect substitutability between digital currency and bank deposits, while the banking sector operates in monopolistic competition. According to the authors, this setup, allied to the availability of external alternative funding for banks, could limit the negative implications of implementing a digital currency. The authors also show that the main benefit of implementing the digital currency would be associated with the wealth redistribution from bank to non-bank agents.

One thing in common across all works mentioned so far is that little attention is paid to the intermediation of CBDC. Some central banks already defined their design of the CBDC service as depending on private partners to provide the interface for final users,[7] with the central bank as the sole issuer of the currency.[8] The private partner would have to be reimbursed for their services, otherwise there would not be any incentive to take part in the venture. The remuneration could come in the form of fees for services in addition to the basic wallet provision (e.g. special interface, budgeting tools etc.).[9]

4 The CBDC intermediation

The model exposed in this section is largely based on that of Cúrdia and Woodford (2016) (hereafter CW). This model is attractive as it contains heterogeneous households which can be of two types: an *impatient* (m), that uses additional financing services

[7]In some cases, the partner would not only have to provide the interface but also be responsible for Anti-Money-Laundry monitoring and Know-Your-Customer policies.

[8]Some cases are the Bank of England digital Pound, the Nigerian E-Naira and the Bahamian Sand-dollar.

[9]See HM Treasury (2023).

with the wallet provider, and a *patient* (p), that holds deposits in CBDC with them. This generates a positive flow of currency between the two different households.

The main difference to CW corresponds to the program of the wallet provider. They have to screen for potential clients at a cost and have a probability associated with services to bad clients. This probability is decreasing in the effort allocated to client monitoring. The productive sector, wage bill and prices are as CW.

4.1 Model setup

4.1.1 Households

The households in this section are analogous to CW. A household i of type $\mathrm{t} \in \{\mathrm{m}, \mathrm{p}\}$ maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left[\frac{1}{1-\sigma_{\mathrm{t}}} c_t(i)^{1-\sigma_{\mathrm{t}}} (\bar{C}_t^{\mathrm{t}})^{\sigma_{\mathrm{t}}} - \int_0^1 \frac{\psi_{\mathrm{t}}}{1+\nu} h_t(i;j)^{1+\nu} dj \right] \tag{4.1}$$

where $c(i)$ is a composite of differentiated goods aggregated following a CES aggregator

$$c_t(i) \equiv \left[\int_0^1 c_t(i;j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} . \tag{4.2}$$

$\bar{C}_t^{\mathrm{t}}$ are exogenous preference shocks and $h_t(i;j)$ is the amount of differentiated labour provided for the production of good j.

In each period, there is a probability $(1-\delta)$ of a new type being drawn by the household, with $\zeta \in (0,1)$ the probability of it being an impatient type. The model assume that the marginal utility of consumption to the impatient type is higher than to the patient type in equilibrium and around its neighbourhood. Furthermore, as assumed in CW, there is an insurance agency that households have intermittent access to every time a new type is drawn, which households can use to insure themselves against aggregate and idiosyncratic risks. This assumption is done to help with aggregation and to avoid dispersion of households' marginal utility of income.

At the end of the period, household i's financial wealth will be

$$B_t(i) = A_t(i) - P_t c_t(i) + \int W_t(j) h(i;j) + D_t + T_t \tag{4.3}$$

where P_t is the price index in period t, $W(j)$ is the wage for labour type j, D_t are the distributed profits from firms that produce differentiated goods and T_t are lump sum government transfers. $A(i)$ is the beginning-of-period financial wealth given by

$$A_t(i) = B_{t-1}(i)(1 + i^{\mathrm{t}}_{t-1}) + D^{\mathrm{CP}}_t + T^s_t(i) \tag{4.4}$$

where $B_t(i) > 0$ implies $i^{\mathrm{t}}_t = i^d_t$ and $i^{\mathrm{t}}_t = i^{\mathrm{f}}_t$ otherwise.[10] D^{CP}_t is the profit from commercial partners' activities distributed back to the households and $T^s_t(i)$ is the transfer between households that have access to the insurance agency.

The total amount of financing services in this economy is given by

[10] For now, the description of the model follows CW, including the fact that deposits in CBDC bear interest. I later restrict this by assuming $i^d_t = 0$.

$$P_t \mathrm{f}_t = -\int_{A_t(i)<0} A_t(i) di \tag{4.5}$$

which is the beginning-of-period wealth summed across all households of type m. Similarly, aggregating across all households of type p yields

$$P_t \mathrm{dc}_t = \int_{A_t(i)>0} A_t(i) di \tag{4.6}$$

which are the aggregate savings in the form of CBDC or government bonds of this economy.[11]

The first-order conditions for each individual household type imply the following Euler equations

$$\lambda_t(i) = \beta(1 + i_t^d)\mathbb{E}_t\left[\frac{\lambda_{t+1}(i)}{\Pi_{t+1}}\right] \qquad \text{(if of type p)}$$
$$\lambda_t(i) = \beta(1 + i_t^{\mathrm{f}})\mathbb{E}_t\left[\frac{\lambda_{t+1}(i)}{\Pi_{t+1}}\right] \qquad \text{(if of type m)}$$

where $\Pi_t \equiv P_t/P_{t-1}$ is the gross inflation in t, and $\lambda_t(i)$ is the Lagrange multiplier for the beginning-of-period budget constraint.

Under the aggregation conditions of the model, each household type has "the same marginal utility of income"(Cúrdia and Woodford, 2016). Hence the Euler equations can be expressed as

[11] This follows the condition in CW that every household of type m will have negative beginning-of-period wealth and every household of type p will have positive beginning-of-period wealth.

$$\lambda_t^{\mathrm{m}} = \beta \mathbb{E}_t \left\{ \frac{1+i_t^{\mathrm{f}}}{1+\Pi_{t+1}} \left[[\delta + (1-\delta)\zeta]\lambda_{t+1}^{\mathrm{m}} + (1-\delta)(1-\zeta)\lambda_{t+1}^{\mathrm{p}} \right] \right\} \tag{4.7}$$

and

$$\lambda_t^{\mathrm{p}} = \beta \mathbb{E}_t \left\{ \frac{1+i_t^{d}}{1+\Pi_{t+1}} \left[(1-\delta)\zeta\lambda_{t+1}^{\mathrm{m}} + [\delta + (1-\delta)(1-\zeta)]\lambda_{t+1}^{\mathrm{p}} \right] \right\} \tag{4.8}$$

where λ_t^i is the marginal utility of consumption for a household type $i \in \{\mathrm{m}, \mathrm{p}\}$.

4.1.2 Wallet provider

The wallet provider collects a specific amount of digital currency dc_t from the households that are willing to withhold consumption until the next period (patient households). For the time being, this deposit-like amount bears interest i_t^d and is a liability to the wallet providers. The latter then offers a financing service f_t used by households that would like to use consumption financing services in the current period (impatient) and are willing to pay for it in the next period.

With probability $(1 - p(\varepsilon_t))$, the partner can incur a fraud cost κ proportional to the amount of consumption financing they provide. Monitoring cost ε_t expends economy and commercial partner's resources and $\varepsilon' > 0$ and $\varepsilon'' \geq 0$. For the effort technology ε_t and probability function I consider the following specifications:

$$\varepsilon_t = \chi \frac{\mathrm{f}_t^{1+\eta}}{1+\eta} \qquad\qquad p(\varepsilon_t) = \frac{1}{1+\exp(-(\beta_0 + \beta_1 \varepsilon_t))}$$

where χ is the unit cost of effort in terms of composite goods, β_0 and β_1 are scaling factors for the logistic function.[12]

Assumption 1 *For the remainder of this paper, I assume that in equilibrium and in*

[12] In appendix A I present an alternative formulation of the problem, where effort is not a technology but a choice variable for the commercial partner.

its neighbourhood the following holds:

(i) The probability function will have the property that $p'' < 0$.

(ii) The wallet provider's problem will result in an interior solution.

Assumption 1 implies that monitoring will have decreasing returns to the wallet provider in terms of decreasing fraud and that there will be a unique solution for the provider's problem. The wallet provider collects CBDC deposits and provides consumption financing services using them as a resource. Any CBDC that is not used for this service is then reverted to the households. Hence, the objective of the commercial partner can be written as

$$\Pi_t^{\mathrm{CP}} = \mathrm{dc}_t - \mathrm{f}_t - (1 - p(\varepsilon_t))\kappa \mathrm{f}_t - \varepsilon(\mathrm{f}_t) \ . \tag{4.9}$$

In the absence of profits when contracts mature, commercial partners provide enough consumption financing service to pay back the CBDC held and the potential loss due to fraudulent users. Hence

$$(1 + i_t^{\mathrm{f}})\mathrm{f}_t = (1 + i_t^d)\mathrm{dc}_t + \kappa \mathrm{f}_t \implies \mathrm{dc}_t = \frac{1 + i_t^{\mathrm{f}}}{1 + i_t^d}\mathrm{f}_t - \frac{\kappa}{1 + i_t^d}\mathrm{f}_t \ . \tag{4.10}$$

Where

$$1 + \omega_t = \frac{1 + i_t^{\mathrm{f}}}{1 + i_t^d} \ .$$

Hence, the fee charged by the commercial partner works as a markup over eventual interest paid in CBDC deposits. Plugging (4.10) into (4.9) yields

$$\Pi_t^{\mathrm{CP}} = \omega_t \mathrm{f}_t - \frac{\kappa}{(1+i_t^d)}\mathrm{f}_t - (1-p(\varepsilon_t))\kappa \mathrm{f}_t - \varepsilon(\mathrm{f}_t) \ . \tag{4.11}$$

The first order condition with respect to f_t implies markup charged by the commercial partner is

$$\omega_t = \frac{\kappa}{(1+i_t^d)} + \varepsilon'(\mathrm{f}_t) + (1-p(\varepsilon_t))\kappa - p'(\varepsilon_t)\varepsilon'(\mathrm{f}_t)\kappa \mathrm{f}_t \ . \tag{4.12}$$

The term $(1-p(\varepsilon_t))\kappa$ in (4.12) corresponds to the marginal expected cost of fraud, while $p'(\varepsilon_t)\varepsilon'(\mathrm{f}_t)\kappa \mathrm{f}_t$ is the marginal expected benefit of monitoring effort. Notice also that the higher interest rate on deposits i_t^d cushions the fraud cost in the first term.

Under the parametrization used in this paper, equation (4.12) implies an increasing and convex function for ω_t with respect to consumption payment services as shown in Figure 2.

4.1.3 Productive sector and closing the model

The only shocks present in the economy are productivity, household consumption preferences, wage markup and monetary policy shocks. The rest of the model follows Cúrdia and Woodford (2016) without any changes.

Household i takes the wage as given and supplies hours of labour $h(i, j)$ to the point that marginal benefit equals marginal disutility of labour. Aggregating across all households for the two types provides the real wage relationship of the form

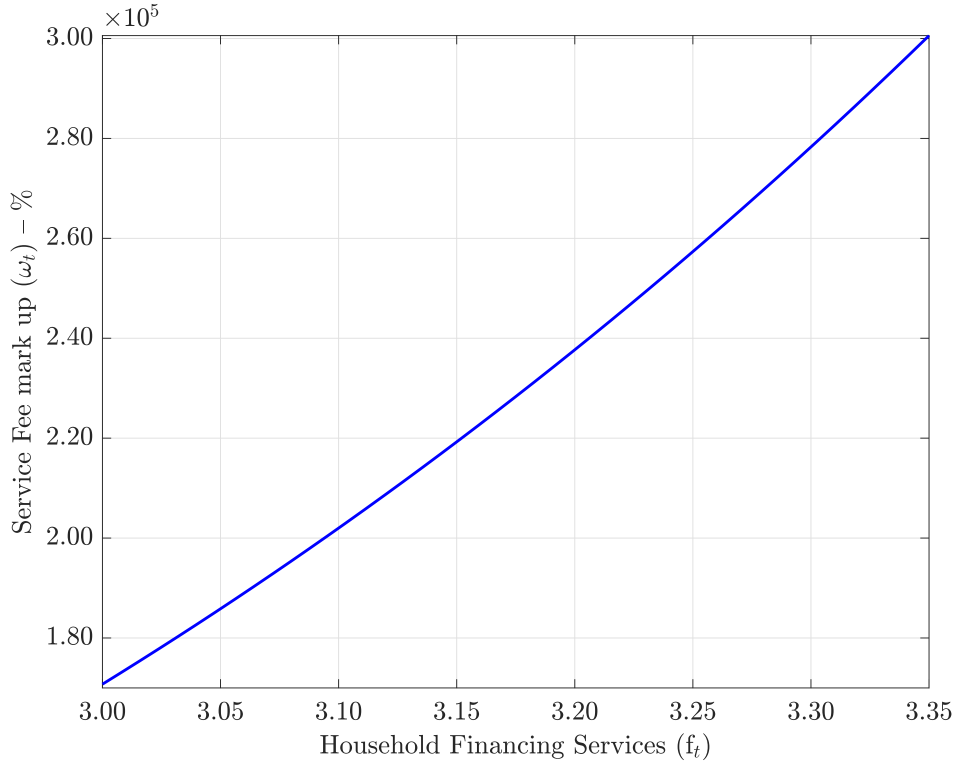


Figure 2: Service fees markup charged by the commercial partner depending on the dimension of consumption payment service issue.

$$W_t(j)/P_t = \mu^w \psi \tilde{\lambda}_t^{-1} (h_t(j))^\nu \tag{4.13}$$

where $W(j)$ is the nominal wage of labour type j, P_t is the price level, μ^w is a markup factor given by potential imperfect competition in the labour market and ν is the inverse Frisch elasticity.

The parameter ψ and the variable $\tilde{\lambda}_t$ are the composites

$$\psi = \left[\zeta\psi_{\mathrm{m}}^{-1/\nu} + (1-\zeta)\psi_{\mathrm{p}}^{-1/\nu}\right]^{\nu}$$
$$\tilde{\lambda}_t = \psi\left[\zeta\left(\frac{\lambda_t^{\mathrm{m}}}{\psi_{\mathrm{m}}}\right)^{\frac{1}{\nu}} + (1-\zeta)\left(\frac{\lambda_t^{\mathrm{p}}}{\psi_{\mathrm{p}}}\right)^{\frac{1}{\nu}}\right]^{\nu}$$

where ψ_{m} and ψ_{p} are scaling parameters on the utility function with respect to the disutility of labour for each household type.

The differentiated goods are produced following the isoelastic function

$$y_t(l) = Z_t h_t(l)^{1/\phi} \tag{4.14}$$

where $\phi \geq 1$ and Z_t is a productivity factor. Preferences for differentiated goods follow a Dixit-Stiglitz aggregator, hence the demand for each good l is given by

$$y_t(l) = Y_t\left(\frac{P_t(l)}{P_t}\right)^{-\theta} \tag{4.15}$$

where Y_t is the demand for the composite good and θ is the elasticity of substitution. Prices are adjusted following Calvo (1983) with probability $(1-\alpha)$.

The resource constraint of the economy is given by

$$Y_t = C_t + G_t + \varepsilon(\mathrm{f}_t) \tag{4.16}$$

where C_t is the aggregate consumption of households of both types, G_t is government expenditure and ε are resources allocated to monitoring clients. Government expenditure is exogenous, follows an AR(1) and is financed by government debt, which is held by patient households, by a proportional tax τ on sales and lump sum tax on income.

4.1.4 Equilibrium

The model can be simplified with the corresponding log-linearized approximations of (4.7),(4.8) and (4.16) around the zero-inflation steady state, obtaining the intertemporal IS curve. Log-linearizing equations (4.7) and (4.8) around the zero-inflation steady state provides

$$\hat{\lambda}_t^{\mathrm{m}} = \hat{i}_t^{\mathrm{f}} - \mathbb{E}_t\pi_{t+1} + \chi_{\mathrm{m}}\mathbb{E}_t\hat{\lambda}_{t+1}^{\mathrm{m}} + (1-\chi_{\mathrm{m}})\mathbb{E}_t\hat{\lambda}_t^{\mathrm{p}} \tag{4.17}$$

and

$$\hat{\lambda}_t^{\mathrm{p}} = \hat{i}_t^{d} - \mathbb{E}_t\pi_{t+1} + (1-\chi_{\mathrm{p}})\mathbb{E}_t\hat{\lambda}_{t+1}^{\mathrm{m}} + \chi_{\mathrm{p}}\mathbb{E}_t\hat{\lambda}_t^{\mathrm{p}} \tag{4.18}$$

where $\chi_{\mathrm{m}} \equiv \beta(1+\bar{i}^{\mathrm{f}})[\delta + (1-\delta)\zeta]$ and $\chi_{\mathrm{p}} \equiv \beta(1+\bar{i}^{d})[\delta + (1-\delta)(1-\zeta)]$. Combining both equations with the log-linearized version of (4.16) obtains the following IS curve

$$\begin{aligned}\hat{Y}_t = -\frac{1}{\bar{\sigma}}(i_t^{\mathrm{avg}} - \mathbb{E}_t\pi_{t+1}) + \mathbb{E}\hat{Y}_{t+1} - s_c\mathbb{E}\Delta\bar{c}_{t+1} - \mathbb{E}\Delta\hat{G}_{t+1} \\ - \mathbb{E}_t\Delta\hat{\varepsilon}_{t+1} - \frac{s_\Omega}{\bar{\sigma}}\hat{\Omega}_t + \frac{(s_\Omega + \psi_\Omega)}{\bar{\sigma}}\mathbb{E}\hat{\Omega}_{t+1}\end{aligned} \tag{4.19}$$

where $\bar{\sigma}, s_\Omega, \psi_\Omega$ are the following composite parameters

$$\frac{1}{\bar{\sigma}} \equiv \zeta s_{\mathrm{m}}\frac{1}{\sigma_{\mathrm{m}}} + (1-\zeta)s_{\mathrm{p}}\frac{1}{\sigma_{\mathrm{p}}}$$

$$s_\Omega \equiv \zeta(1-\zeta)\frac{s_{\mathrm{m}}/\sigma_{\mathrm{m}} - s_{\mathrm{p}}/\sigma_{\mathrm{p}}}{1/\bar{\sigma}}$$

$$\psi_\Omega \equiv \zeta(1-\chi_{\mathrm{m}}) - (1-\zeta)(1-\chi_{\mathrm{p}})$$

and $i_t^{\text{avg}}, \bar{c}_t, \hat{\Omega}_t$ are the average of interest rates from consumption payment services and CBDC holdings, a measure of change in aggregate consumption preferences and a measure of inefficiency in the intermediation of CBDC given by

$$\hat{i}_t^{\text{avg}} \equiv \zeta \hat{i}_t^{\text{f}} + (1-\zeta)\hat{i}_t^{d} \implies \hat{i}_t^{\text{avg}} \equiv \hat{i}_t^{d} + \zeta\omega_t$$

$$s_c\bar{c}_t \equiv \zeta s_{\text{m}}\bar{c}_t^{\text{m}} + (1-\zeta)s_{\text{p}}\bar{c}_t^{\text{p}}$$

$$\hat{\Omega}_t \equiv \hat{\lambda}_t^{\text{m}} - \hat{\lambda}_t^{\text{p}} \ .$$

The parameters $s_c, s_{\text{m}}, s_{\text{p}}$ correspond to the share of consumption in the total expenditure at the steady state and the share of consumption of each household type ($s_c = C/Y$ and $s_i = C_i/Y$ for $i \in \{\text{m}, \text{p}\}$). $\sigma_i, i \in \{\text{m}, \text{p}\}$ is the intertemporal elasticity of substitution for each household type.

From equations (4.17) and (4.18) we can obtain the following forward-looking equation for Ω_t

$$\hat{\Omega}_t = \hat{\omega}_t + \hat{\delta}\mathbb{E}_t\hat{\Omega}_{t+1} \tag{4.20}$$

with $\hat{\delta} = \chi_{\text{m}} + \chi_{\text{p}} - 1$.

The aggregate supply curve, considering the simplification mentioned above, is given by

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + s_\pi(\hat{Y}_t - \hat{Y}_t^n) + \xi(s_\Omega + \zeta - \gamma_{\text{m}})\hat{\Omega}_t - \xi(\bar{\sigma}\hat{\varepsilon}_t - \hat{\mu}_t^w) \tag{4.21}$$

where the slope is given by $s_\pi \equiv \xi(\omega_y + \bar{\sigma})$ and the composite parameters $\omega_y, \xi, \gamma_{\mathrm{m}}$ are

$$\omega_y \equiv \phi(1-\nu) - 1$$
$$\xi \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\omega_y\theta)}$$
$$\gamma_{\mathrm{m}} \equiv \zeta \left(\frac{\psi\bar{\lambda}^{\mathrm{m}}}{\psi_{\mathrm{m}}\bar{\bar{\lambda}}}\right)^{\frac{1}{\nu}} .$$

The term $\hat{Y}_t^n$ in (4.21) corresponds to the aggregate exogenous disturbances in consumption preferences and productivity, and can be considered as the natural rate of output, or the "variations in the efficient level of output "(Cúrdia and Woodford, 2016) and is given by

$$\hat{Y}_t^n = (\omega_y + \bar{\sigma})^{-1} \left[\bar{\sigma}(s_c \bar{c}_t + \hat{G}_t) + (1+\omega_y) z_t\right] \tag{4.22}$$

with z_t the log change in the productivity factor.

Finally, the log-linearized version of (4.12) is given by

$$\hat{\omega}_t = \omega_i \hat{i}_t^d + \omega_f \hat{\mathrm{f}}_t \tag{4.23}$$

with

$$\omega_i = -\frac{\kappa}{(1+\bar{\omega})(1+\bar{i}^d)}$$
$$\omega_{\mathrm{f}} = \frac{\bar{\mathrm{f}}}{(1+\bar{\omega})}\left[\bar{\varepsilon}'' - \bar{p}'\bar{\varepsilon}'\kappa - \left(\bar{p}''\bar{\varepsilon}'^2\kappa\bar{\mathrm{f}} + (\varepsilon''\kappa\bar{\mathrm{f}} + \varepsilon'\kappa)\bar{p}'\right)\right] \ .$$

Equation (4.23) depends on the level of consumption financing services in this economy for which the log linearized function is as CW, given by

$$\begin{aligned}\hat{\mathrm{f}}_t &= \varrho_r\left(\hat{i}^d_{t-1} - \pi_t\right) + \varrho_Y\hat{Y}_t + \varrho_\Omega\hat{\Omega}_t + \varrho_\omega\hat{\omega}_t + \varrho_{\mathrm{f}}\left(\hat{\mathrm{f}}_{t-1} + \hat{\omega}_{t-1}\right)\\ &+ \varrho_\xi\left[\zeta\,(1-\zeta)\,s_c\bar{c}_t + B_\lambda\bar{\sigma}\left(s_c\bar{c}_t + \hat{G}_t + \hat{\varepsilon}_t\right) - B_u\left[\hat{\mu}^w_t - (1+\omega_y)\,z_t\right]\right]\\ &\qquad - \zeta\varrho_\xi\left[\hat{b}^g_t - \delta\left(1+\bar{i}^d\right)\hat{b}^g_{t-1}\right]\end{aligned} \quad (4.24)$$

where ϱ_x are composite parameters from the log-linearization. (4.23) also depends on the deposit rate, which is given by the central bank policy function

$$\hat{i}^d_t = \phi_p\pi_t + \phi_y\hat{Y}_t + \epsilon^i_t \quad (4.25)$$

where ϵ^i_t are monetary policy surprises.

Equations (4.19) to (4.25) then determine the equilibrium given by the set of variables $\{\hat{Y}_t, \hat{\Omega}_t, \pi_t, \hat{Y}_t, \hat{\omega}_t, \hat{\mathrm{f}}_t, \hat{i}^d_t\}$.

In the next section, I present the simulation results considering three versions of the model: (1) a version where the central bank targets zero inflation rate, (2) a version that incorporates the fact that CBDC does not bear interest, and (3) a version without the financial frictions.

5 Results

In this section, I present the IRFs of selected variables with respect to, TFP, consumption preferences and wage mark-up shocks under three model specifications. The first specification (labelled "Zero Inflation" in the diagrams) is the model considering a zero inflation target where the output is defined by rewriting equation (4.21) as

$$\hat{Y}_t = Y_t^n - \frac{\xi}{s_\pi}(s_\Omega + \zeta - \gamma_b)\hat{\Omega}_t + \frac{\xi}{s_\pi}(\bar{\sigma}^{-1}\hat{\varepsilon}_t - \hat{\mu}_t^w) \tag{5.1}$$

which relation implicitly defines an interest rate i_t^d that affects the household financing evolution given by equation (4.24).

The second specification of the model (labelled "No Interest") incorporates into the zero-inflation model, the constraint that CBDC deposits bear no interest. Because the friction faced by the wallet provider is still present, they will still charge a markup to provide the consumption services. The final specification (labelled "No Frictions") then turns off the financial frictions. Under the latter specification, there is no probability of misuse of consumption financing services and there's no cost associated with proving them as well. Hence, the markup rate ω_t and the monitoring cost ε_t will both be equal at all times. For the results presented in this section, I use the parameters presented in Table 1 which were parametrised as in CW.

Figure 3, shows the response of a TFP shock. One striking distinction across specifications is that part of the dynamics on the variables associated with the CBDC intermediation shows opposing dynamics when comparing the zero-inflation to the zero-interest specifications. In the zero-inflation specification, the central bank will coun-

Table 1: Model Paramaters

Parameter	Description	Value	Parameter	Description	Value
β	Discount Factor	0.989	$1-\delta$	Probability of Drawing a New Type	0.025
ζ	Share of Impatient households	0.500	s_c	Consumption share of Composite Good	0.700
s_{m}	Share of Impatient Consumption	0.886	s_{p}	Share of Patient Consumption	0.618
σ^{m}	Intertemporal Elasticity – Impatient Household	0.400	σ^{p}	Intertemporal Elasticity – Patient Household	1.500
α	Calvo Factor	0.660	ω_y	Wage bill composite	0.473
θ	Elasticity of Substitution Across Goods	7.667	ν	Inverse Frisch Elasticity	0.105
$1/\phi$	Labour Share	0.750	ψ	Composite Scaling Parameter for Labour supply	1.000
$\bar{\imath}^d$	Steady State Interest Rate	0.010	$\bar{\omega}$	Steady State Markup Rate	0.025
β_0	Logistic Scale Parameter	3.000	β_1	Logistic Scale Parameter	15.00
$\bar{\mathrm{f}}$	Steady State Household Financing	3.200	κ	Unit of Fraud Cost	0.030
η	Effort Elasticity of Household Financing	13.60	$\log(\chi)$	Log Unit Price of Monitoring	-11.46
ϕ_π	Inflation Elasticity on Taylor Rule	1.500	ϕ_y	Output Elasticity on Taylor Rule	0.250
ρ_i	AR(1) coefficient for Mon. Policy Shock	0.600	$\bar{\mu}^w$	Steady State Labour Market Markup	1.000
ρ_c	AR(1) coefficient for Cons. Preference Shock	0.900	σ_i	Variance of Mon. Policy Shock	0.063
$\sigma_{c^{\mathrm{p}}}$	Variance of Cons. Preference Shock – Patient Household	4.300	$\sigma_{c^{\mathrm{m}}}$	Variance of Cons. Preference Shock – Impatient houshehold	10.00
ρ_g	AR(1) coefficient for Gov. Expenditure Shock	0.900	σ_g	Variance of Gov. Expenditure Shock	1.000
ρ_{μ^w}	AR(1) coefficient for Labour Market Shock	0.900	σ_{μ^w}	Variance of Labour Market Shock	1.000
ρ_z	AR(1) coefficient for TFP Shock	0.900	σ_z	Variance of TFP Shock	1.000

Note: The parameters were set as in Cúrdia and Woodford (2016) with a higher steady state value of household markup rate ($\bar{\omega}$). The parameters are the same under the three specifications of the model.

teract the positive TFP shock by increasing interest rates, which has two unfolding. Firstly, the higher interest rate attracts more patient households' savings that could be used to generate more consumption financing services for impatient households. It also scares away impatient households' demand for consumption financing services, given an increase in the cost.

Secondly, an increase in income can make impatient households less reliant on consumption finance services to pay for their consumption, reducing their demand for such. Figure 3 suggests that the latter effect is stronger on impact under a zero-inflation policy, forcing the wallet provider to absorb part of the interest rate increase, as they see the demand for consumption financing services decreasing. However, as the shock fades away, the demand for consumption financing services picks up and the wallet provider can pass through the cost again to the final user, raising the markup.

When the CBDC is constrained to bear no interest, the mechanism described above is less present. Patient households have a lower incentive to keep deposits in CBDC, hence there is less inflow of resources to the wallet providers. The reduction in demand for consumption financing is, however, more persistent and it doesn't present the rebound as described before. Because CBDC deposits do not pay interest, the wallet provider can reduce by a lower amount the markup rate, given the reduction in demand for consumption financing services.

When frictions are removed from the model, TFP shocks result in a lasting decrease in the consumption financing services. Since wallet providers lack mechanisms to mitigate the decline in demand for their services, a reduction in savings from patient households leads to fewer resources available to these wallets. Combined with an increase in income during the current period, impatient households will be last dependent on wallet services too.

Figure 4 presents the impact of a change in consumption preferences for type p household. It implies greater consumption in the current period and therefore a reduction in CBDC deposit holdings. One striking distinction is that, while the mechanism is similar to the one exposed above for the zero-inflation specification, when CBDC does not bear interest we obtain different dynamics than before. With zero interest on CBDC deposits, a consumption shock will marginally drive up consumption financing services, and monitoring costs to the wallet provider. This will pass the cost by increasing the markup rate to the household. The persistence of consumption financing services in the frictionless specification is still present in this case. However, notice that the dynamic of output is volatile.

Figure 5 serves as a robustness check and presents the impact of a cost-push shock, an increase in the wage markup. The dynamics of the variables are in the opposite direction of the TFP shock, with the transition mechanism being analogous to that case.

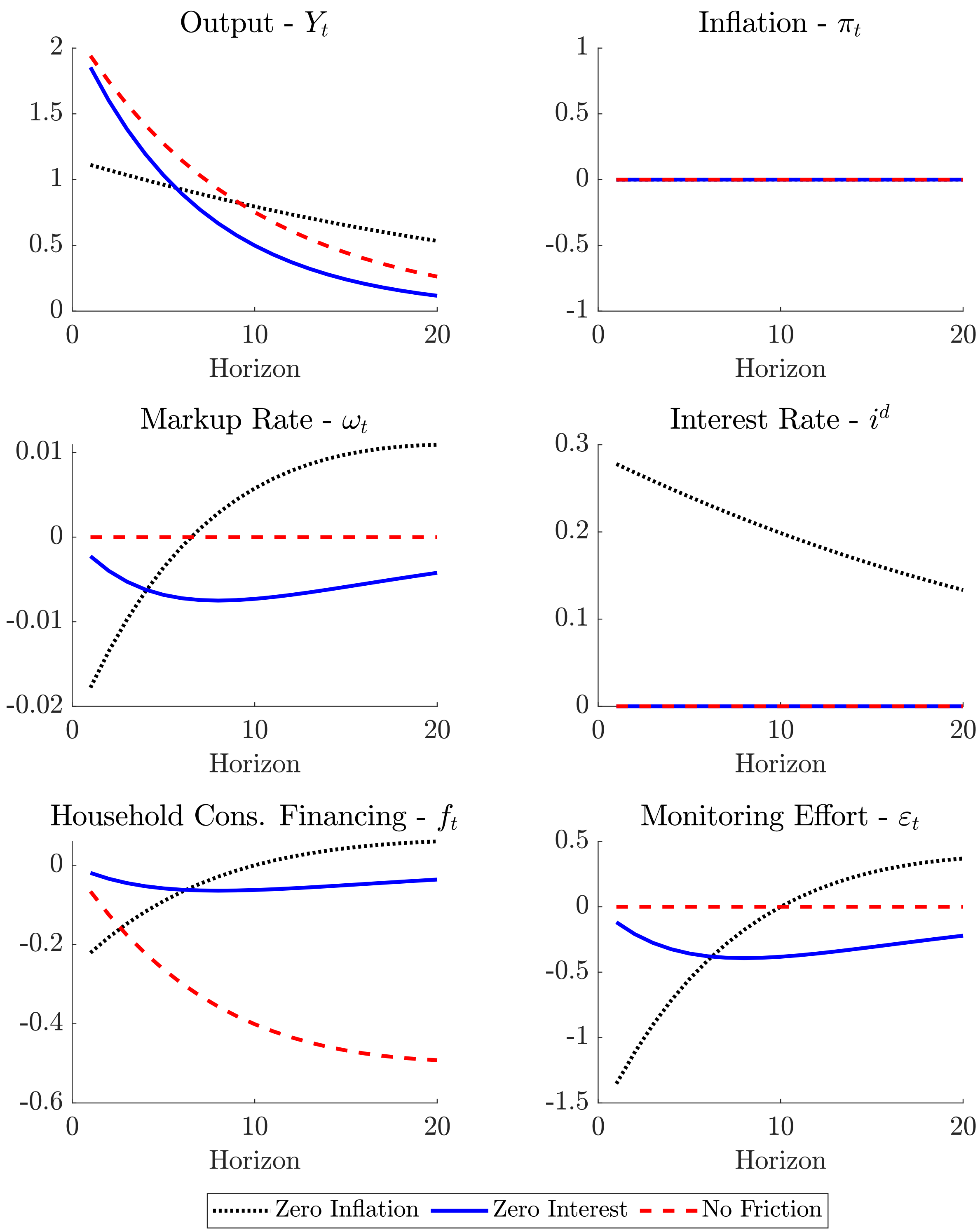


Figure 3: Impulse response to positive TFP shock for selected variables.

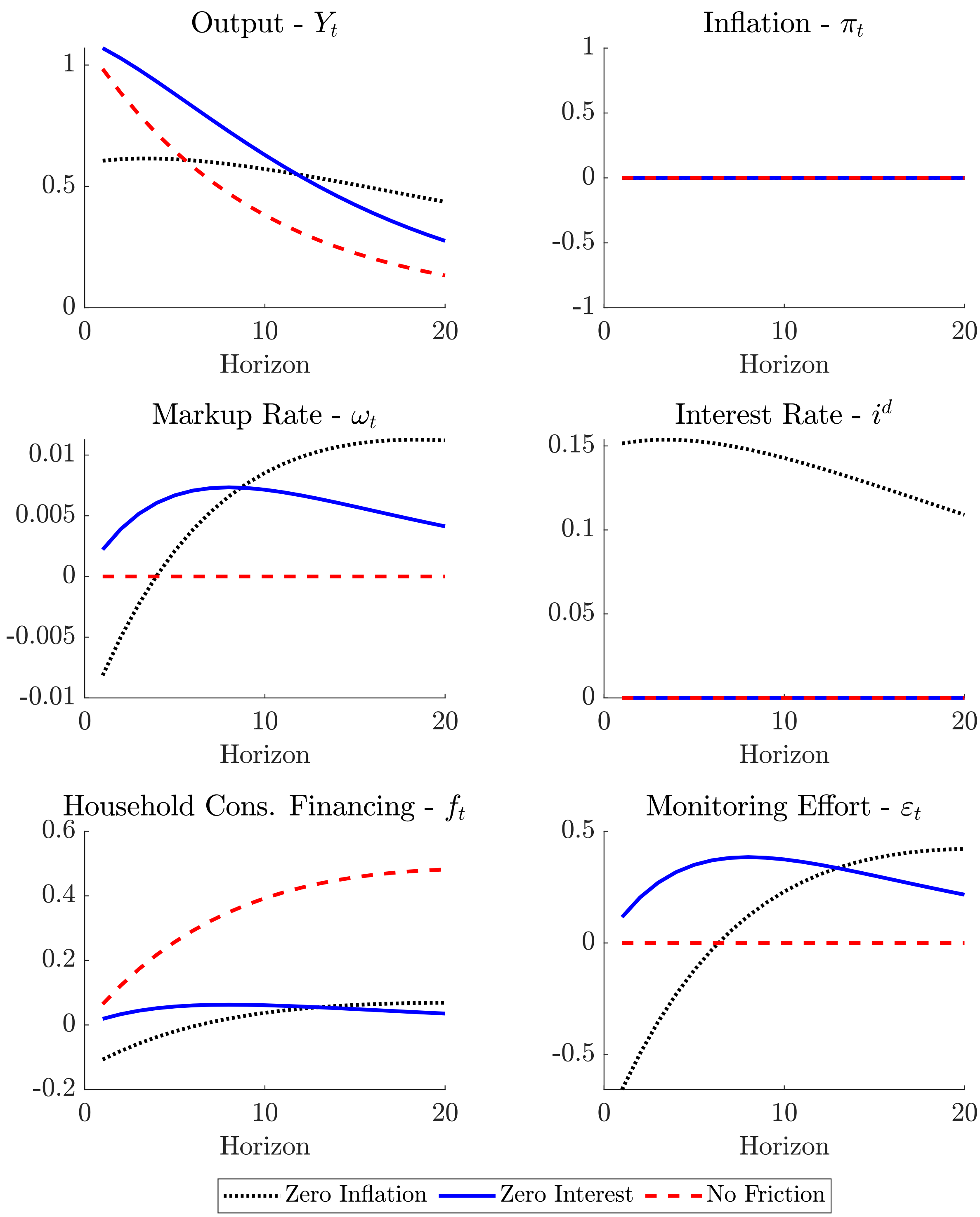


Figure 4: Impulse response to consumption preference for selected variables.

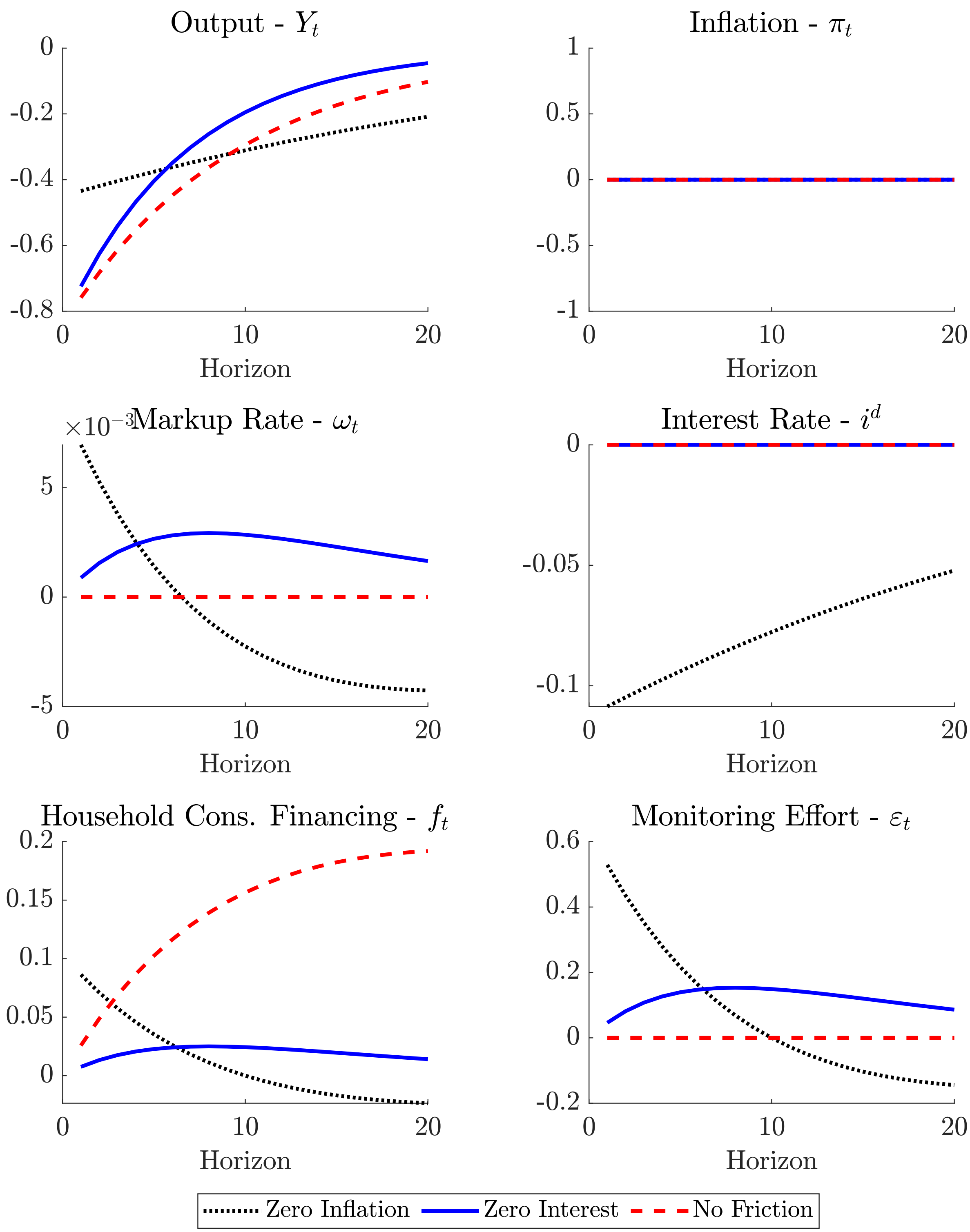


Figure 5: Impulse response to cost-push shocks for selected variables

6 Discussion

This paper investigates the role of wallet providers in the intermediation of Central Bank Digital Currencies (CBDC), extending to their role in providing consumption financing services to households. The paper uses the model of Cúrdia and Woodford (2016) to consider monitoring cost by the wallet, the dynamics of the services provision and its markup fees. The main takeaway from the analysis is that interest rate dynamics and the potential of charging service fees play a significant role in the dynamics of the services' demand. If central banks restrict CBDC deposits to bear no interest, fewer patient households will hold these deposits and wallet providers will have fewer resources to generate consumption financing services. In the absence of financial frictions, the wallet services have a stronger persistency to shocks than in other specifications.

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A Choosing Effort

If the commercial partner were allowed to choose optimal effort instead of it being a function of the amount of financing service, the problem given by (4.11) can be re-writen as

$$\Pi_t^{\mathrm{CP}} = \omega_t \mathrm{f}_t - \frac{\kappa}{(1+i_t^d)}\mathrm{f}_t - (1-p(\varepsilon_t))\kappa \mathrm{f}_t - \frac{c}{2}\varepsilon_t^2 \tag{A.1}$$

The first-order conditions are then

$$[\mathrm{f}_t] \qquad \omega_t - \frac{\kappa}{1+i_t^d} - (1-p(\varepsilon_t))\kappa = 0 \tag{A.2}$$

$$[\varepsilon_t] \qquad p'(\varepsilon_t)\frac{\kappa}{c}\mathrm{f}_t - \varepsilon_t = 0 \tag{A.3}$$

In this specification, equation (A.3) implies that effort is increasing in the consumption financing services as before. However, note that (A.2) implies that effort only affects the markup via its impact on the probability of a bad client. Given that the probability is decreasing in effort, the new specification implies that the markup will now have a negative relation with respect to the amount of consumption financing services provided as illustrated in Figure A.1.

One way to rationalise this result is that because the wallet provider needs to increase monitoring of their portfolio of clients, given an increase in consumption fi-

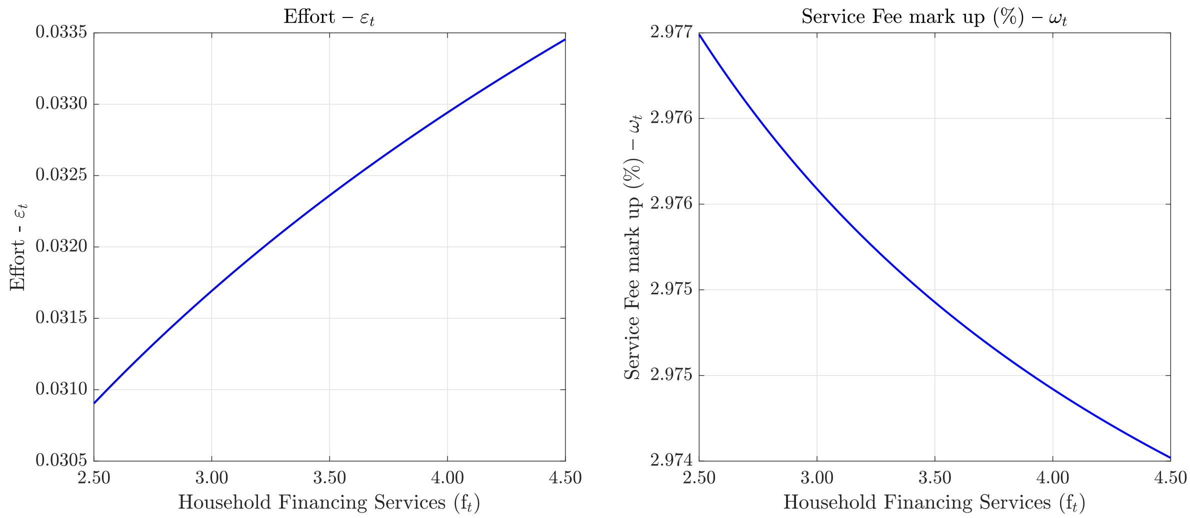

Figure A.1: Alternative specification for service fees charged by commercial partner

nancing services, they can do a better job in screening for better clients, reducing the probability of fraud. This in turn is passed through to the final user by charging a lower markup on their services.